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PURE MATHEMATICS 10–20–30

Program of Studies

Alberta
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PURE MATHEMATICS 10–20–30

A. PROGRAM RATIONALE AND PHILOSOPHY

Pure mathematics emphasizes mathematical theory and the testing of mathematical hypotheses. The pure mathematics approach, which is often deductive and symbolic, endeavours to show that concepts are valid all the time, or valid within a well-defined set of restrictions. Real-life problems are then presented in order for students to apply previously learned mathematical concepts and procedures. Students will make use of algebra and graphing to solve problems. Algebra is taught both on a "when needed basis" and for its own sake.

Students are required to demonstrate effective communication skills. This includes understanding, using, and interpreting various mathematical concepts and processes. When accomplishing program or curriculum outcomes, students will be expected to explain, to illustrate, to reason and to make connections. Multiple solution strategies to problems and problem contexts will be expected as students work through both routine and nonroutine problems.

Students will be expected to take responsibility for their learning as they work both independently and in groups. Tasks vary from short procedural items that help students develop skills in the language of mathematics, to longer tasks that require students to test either subtle conjectures or apply mathematical knowledge to nonroutine, real-life problems.

Technology is a part of pure mathematics. The graphing calculator is the primary technological tool used by students for mathematical exploration, modelling and problem solving. The use of spreadsheets, with functions defined by the student, can be profitable in many contexts.

The Pure Mathematics 10–20–30 program is made up of outcomes, some of which are common to both the pure and applied mathematics programs. The approach to the common outcomes varies from the pure to the applied programs, but the critical skill of using mathematics to find solutions to real-life situations is developed in both programs.

B. LEARNER OUTCOMES

The general outcomes for the complete Pure Mathematics 10–20–30 program are shown first. For Pure Mathematics 10 and Pure Mathematics 20, the learner outcomes follow the format of general outcome and specific outcomes. For Pure Mathematics 30, the learner outcomes follow the format of general outcome, specific outcomes and illustrative examples. The illustrative examples are for discretionary use only. The outcomes are arranged in order of strand and substrand, following the order—Number, Patterns and Relations, Shape and Space, Statistics and Probability—established in the Grade 10 to Grade 12 common curriculum framework. This listing is not intended as a sequence for instruction, and the number of outcomes is not an indication of the relative times to be spent on each of the strands. Specific outcomes in Pure Mathematics 30 with a **C** prefix, such as C6–3, refer to those outcomes that form part of the common core, while specific outcomes with a **P** prefix, such as P7–17, refer to those outcomes that are exclusive to pure mathematics.

GENERAL OUTCOMES—Number Strand

Substrand	Pure Mathematics 10–20–30
Number Concepts *Students will:* • use numbers to describe quantities • represent numbers in multiple ways.	Analyze the numerical data in a table for trends, patterns and interrelationships. [Pure Mathematics 10] Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system. [Pure Mathematics 10]
Number Operations *Students will:* • demonstrate an understanding of and proficiency with calculations • decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.	Use basic arithmetic operations on real numbers to solve problems. [Pure Mathematics 10] Describe and apply arithmetic operations on tables to solve problems, using technology as required. [Pure Mathematics 10] Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. [Pure Mathematics 10] Solve consumer problems, using arithmetic operations. [Pure Mathematics 20]

GENERAL OUTCOMES—Patterns and Relations Strand

Substrand	Pure Mathematics 10–20–30
Patterns *Students will:* • use patterns to describe the world and to solve problems.	Generate and analyze number patterns. [Pure Mathematics 10] Apply the principles of mathematical reasoning to solve problems and to justify solutions. [Pure Mathematics 20] Generate and analyze exponential patterns. [Pure Mathematics 30]
Variables and Equations *Students will:* • represent algebraic expressions in multiple ways.	Generalize operations on polynomials to include rational expressions. [Pure Mathematics 10] Represent and analyze situations that involve expressions, equations and inequalities. [Pure Mathematics 20] Solve exponential, logarithmic and trigonometric equations and identities. [Pure Mathematics 30]
Relations and Functions *Students will:* • use algebraic and graphical models to generalize patterns, make predictions and solve problems.	Examine the nature of relations with an emphasis on functions. [Pure Mathematics 10 and 20] Represent data, using linear function models. [Pure Mathematics 10] Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate. [Pure Mathematics 20] Represent and analyze exponential and logarithmic functions, using technology as appropriate. [Pure Mathematics 30] Represent and analyze trigonometric functions, using technology as appropriate. [Pure Mathematics 30]

GENERAL OUTCOMES—Shape and Space Strand

Substrand	Pure Mathematics 10–20–30
Measurement *Students will:* • describe and compare everyday phenomena, using either direct or indirect measurement.	Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. [Pure Mathematics 10] Solve problems involving triangles, including those found in 3-D and 2-D applications. [Pure Mathematics 10 and 20]
3-D Objects and 2-D Shapes *Students will:* • describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	Solve coordinate geometry problems involving lines and line segments. [Pure Mathematics 10] Solve coordinate geometry problems involving lines and line segments, and justify the solutions. [Pure Mathematics 20] Develop and apply the geometric properties of circles and polygons to solve problems. [Pure Mathematics 20] Classify conic sections, using their shapes and equations. [Pure Mathematics 30]
Transformations *Students will:* • perform, analyze and create transformations.	Perform, analyze and create transformations of functions and relations that are described by equations or graphs. [Pure Mathematics 30]

GENERAL OUTCOMES—Statistics and Probability Strand

Substrand	Pure Mathematics 10–20–30
Data Analysis *Students will:* • collect, display and analyze data to make predictions about a population.	Implement and analyze sampling procedures, and draw appropriate inferences from the data collected. [Pure Mathematics 10]
Chance and Uncertainty *Students will:* • use experimental or theoretical probability to represent and solve problems involving uncertainty.	Use normal and binomial probability distributions to solve problems involving uncertainty. [Pure Mathematics 30] Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. [Pure Mathematics 30] Model the probability of a compound event, and solve problems based on the combining of simpler probabilities. [Pure Mathematics 30]

PURE MATHEMATICS 10

Topic 1 – Sequences and Data Tables

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

General and Specific Outcomes

General Outcome	Specific Outcomes
Analyze the numerical data in a table for trends, patterns and interrelationships.	1.1 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are not related recursively (not calculated from previous data). [C, CN] 1.2 Use words and algebraic expressions to describe the data and the interrelationships in a given table with rows that are related recursively (calculated from previous data). [C, CN]
Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.	1.3 Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R] 1.4 Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R]
Describe and apply arithmetic operations on tables to solve problems, using technology as required.	1.5 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V] 1.6 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]
Generate and analyze number patterns.	1.7 Generate number patterns exhibiting arithmetic growth. [E, R] 1.8 Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T] 1.9 Generate number patterns exhibiting geometric growth. [E, R]

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

Topic 2 – Algebraic Expressions

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Generalize operations on polynomials to include rational expressions.	2.1 Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E] 2.2 Find the product of polynomials. [E, R] 2.3 Divide an integral polynomial by a binomial, and express the result in the forms: • $\frac{P}{D} = Q + \frac{R}{D}$ • $P = DQ + R$ • $P(x) = D(x)Q(x) + R$. [E, R] 2.4 Determine equivalent forms of single-variable rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R] 2.5 Determine the nonpermissible values for the variable in single-variable rational expressions [C, CN] 2.6 Perform the operations of addition, subtraction, multiplication and division on single-variable rational expressions.[E, R] 2.7 Find and verify the solutions of rational equations that reduce to linear equations. [CN, PS]

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

Topic 3 – Line Segments and Graphs

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Examine the nature of relations with an emphasis on functions.	3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]
Solve coordinate geometry problems involving lines and line segments.	3.2 Solve problems involving distances between points in the coordinate plane. [PS, V]
	3.3 Solve problems involving midpoints of line segments. [PS]
	3.4 Solve problems involving rise, run and slope of line segments. [PS, V]
	3.5 Determine the equation of a line, given information that uniquely determines the line. [PS, V]
	3.6 Solve problems using slopes of: • parallel lines • perpendicular lines. [CN, PS, V]

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

Topic 4 – Relations and Functions

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Examine the nature of relations with an emphasis on functions.	4.1 Represent data, using function models. [CN, PS, V]
	4.2 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]
	4.3 Describe a function in terms of: • ordered pairs • a rule, in word or equation form • a graph. [C, CN, V]
	4.4 Use function notation to evaluate and represent functions. [C, PS]
	4.5 Determine the domain and range of a relation from its graph. [PS, V]
	4.6 Determine the following characteristics of the graph of a linear function, given its equation: • intercepts • slope • domain • range. [PS, V]
Represent data, using linear function models.	4.7 Use direct variation, partial variation, and arithmetic sequences as applications of linear functions. [CN, PS, V]
Generate and analyze number patterns.	4.8 Relate arithmetic sequences to linear functions defined over the natural numbers. [CN]

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

Topic 5 – Exponents and Radicals

General Outcome	**Specific Outcomes**
Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.	5.1 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V] 5.2 Use approximate representations of irrational numbers. [R, T]
Use basic arithmetic operations on real numbers to solve problems.	5.3 Communicate a set of instructions used to solve an arithmetic problem. [C] 5.4 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]
Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.	5.5 Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E] 5.6 Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

Topic 6 – Measurement and Trigonometry

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Solve problems involving triangles, including those found in 3-D and 2-D applications.	6.1 Solve problems involving two right triangles. [CN, PS, V] 6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V] 6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

PURE MATHEMATICS 20

Topic 1 – Linear and Nonlinear Systems

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Represent and analyze situations that involve expressions, equations and inequalities.	1.1 Graph linear inequalities, in two variables. [PS, V] 1.2 Solve systems of linear equations, in two variables: • algebraically (elimination and substitution) • graphically. [CN, PS, T, V] 1.3 Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V] 1.4 Solve systems of linear equations, in three variables: • algebraically • with technology. [CN, PS, T, V]

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

Topic 2 – Quadratic Functions and Equations

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	2.1 Determine the following characteristics of the graph of a quadratic function: • vertex • domain and range • axis of symmetry • intercepts. [C, PS, T, V] 2.2 Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V] 2.3 Model real-world situations, using quadratic functions. [CN, PS] 2.4 Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: • factoring • the quadratic formula • graphing. [CN, E, T, V] 2.5 Determine the character of the real and non-real roots of a quadratic equation, using: • the discriminant in the quadratic formula • graphing. [C, R, T, V]

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

Topic 3 – Polynomial and Other Nonlinear Equations and Functions

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Represent and analyze situations that involve expressions, equations and inequalities.	3.1 Solve nonlinear equations: • by factoring • graphically. [CN, T, V] 3.2 Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]
Examine the nature of relations with an emphasis on functions.	3.3 Perform operations on functions and compositions of functions. [CN, E, PS] 3.4 Determine the inverse of a function. [CN, R, V]
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	3.5 Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V] 3.6 Formulate and apply strategies to solve: • absolute value equations • radical equations • rational equations • quadratic inequalities • polynomial inequalities. [CN, R, T, V]

C	Communication	PS	Problem Solving
CN	Connections	R	Reasoning
E	Estimation and Mental Mathematics	T	Technology
		V	Visualization

Topic 4 – Formal Reasoning

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Apply the principles of mathematical reasoning to solve problems and to justify solutions.	4.1 Differentiate between inductive and deductive reasoning. [CN, R] 4.2 Explain and apply connecting words, such as "and", "or" and "not", to solve problems. [C, PS, R, V] 4.3 Use examples and counterexamples to analyze conjectures. [CN, R] 4.4 Distinguish between an "if–then" proposition, its converse and its contrapositive. [CN, R] 4.5 Prove assertions in a variety of settings, using direct reasoning. [R]

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

Topic 5 – Circles and Coordinate Geometry

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Develop and apply the geometric properties of circles and polygons to solve problems.	5.1 Use technology and measurement to confirm and apply the following properties to particular cases: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • the angle inscribed in a semicircle is a right angle • the opposite angles of a cyclic quadrilateral are supplementary • a tangent to a circle is perpendicular to the radius at the point of tangency • the tangent segments to a circle, from any external point, are congruent • the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord • the sum of the interior angles of an *n*-sided polygon is (2*n* – 4) right angles. [PS, R, T, V] 5.2 Prove the following general properties, using established concepts and theorems: • the perpendicular bisector of a chord contains the centre of the circle • the angle inscribed in a semicircle is a right angle • the tangent segments to a circle from any external point are congruent 5.3 Solve problems, using a variety of circle properties and relevant trigonometric ratios, and justify the solution strategy used. [PS, R, V]
Solve coordinate geometry problems involving lines and line segments, and justify the solutions.	5.4 Solve problems involving distances between points and lines. [CN, PS, R] 5.5 Verify and prove assertions in plane geometry, using coordinate geometry and trigonometric ratios as necessary. [C, R, V]

C	Communication	**PS**	Problem Solving
CN	Connections	**R**	Reasoning
E	Estimation and Mental Mathematics	**T**	Technology
		V	Visualization

Topic 6 – Finance

General and Specific Outcomes	
General Outcome	**Specific Outcomes**
Solve consumer problems, using arithmetic operations.	6.1 Solve consumer problems, that may include such examples as: • wages earned in various situations • property taxation • exchange rates [CN, E, PS, R, T] 6.2 Reconcile financial statements that may include such examples as: • cheque books with bank statements • credit card or loan statements. [CN, PS, T] 6.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V] 6.4 Plot and describe financial data of exponential form. [C, T, V] 6.5 Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Generate and analyze exponential patterns.

Specific Outcomes

P6–1. Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems.
[CN, R, T]

Illustrative Examples [Discretionary]

1.1 Determine the n^{th} term and the sum of the first n terms of the geometric sequence whose first three terms are 2, 6 and 18.

1.2 Mathematicians use sigma notation as a way to write the sum of a series. For example: $\sum_{k=1}^{5} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$. Use sigma notation to write the series $5 - 15 + 45 - \ldots + 3645$.

1.3 Suppose that a principal of P dollars is invested at an annual interest rate r that is compounded annually. The amount A after t years is given by $A = P(1 + r)^t$.
a) Find the number of years for the amount to double, if $2000 is invested at a rate of 7.5%, compounded annually.
b) If the interest rate were 7.25% per annum, compounded semi-annually, how would the doubling period change?
c) What would be the doubling period, if the rate were 7% per annum, compounded daily?

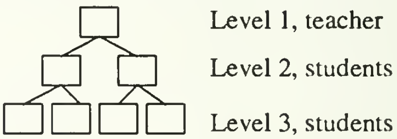
1.4 For the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$, find the sum of 20 terms.

1.5 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years.
a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates:
  - 4% per annum, compounded annually
  - 8% per annum, compounded annually
  - 24% per annum, compounded annually.

b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations?

Strand: Patterns and Relations (Patterns)
Students will:
- use patterns to describe the world and to solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P6–2. Connect geometric sequences to exponential functions over the natural numbers. [E, R, V]	2.1 The world's population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. a) Calculate the population in 1998, 2009, 2019. b) Calculate the population that food production could sustain in 1998, 2009, 2019. c) When will the population exceed the food supply? 2.2 The following is a school trip telephoning tree.  a) At what level are 64 students contacted? b) How many are contacted at the 8th level? c) By the 8th level how many students, in total, have been contacted? d) By the *n*th level how many students, in total, have been contacted? e) If there are 300 students in total, by what level will all have been contacted?
P6–3. Estimate values of expressions for infinite geometric processes. [PS, R, T]	3.1 For the infinite series $2+\frac{2}{5}+\frac{2}{25}+\dots$, estimate the sum to four decimal places. 3.2 An oil well produces 25 000 barrels of oil during its first month of production. If its production drops by 5% each month, estimate the total production before the well runs dry.

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Solve exponential, logarithmic and trigonometric equations and identities.	
Specific Outcomes	
P6–4. Solve exponential equations having bases that are powers of one another. [E, R]	4.1 Solve for x: $3^{(4x-1)} = 27^{2x}$. 4.2 A string of ones and zeros is the binary representation of a number. If this number is converted to the base-16 hexadecimal representation, it is 9 digits shorter. As well, the decimal and hexadecimal representations have the same number of digits. a) How many digits are there in the binary representation of the original number? b) Between what two decimal numbers does the original number lie?
P6–5. Solve and verify exponential and logarithmic equations and identities. [R]	5.1 Solve for x: $\log_2(x-2) + \log_2(x) = \log_2(3)$. 5.2 Solve for x: $2 \times 3^x = 5^{(x-1)}$. 5.3 Solve for x, checking for any extraneous solutions: $\log_5(3x+1) + \log_5(x-3) = 3$. 5.4 The pH of an acid is given by $\mathrm{pH} = -\log_{10}[\mathrm{H}^+]$, where $[\mathrm{H}^+]$ is the hydrogen ion concentration in moles per litre. What is the hydrogen ion concentration of a weak vinegar solution of pH = 3.1? 5.5 Joe has $50 000 invested at an interest rate of 7% per annum, compounded monthly. Laura has $40 000 invested at 9.5% per annum, compounded annually. After how many years will the two investments be equal in value? 5.6 Verify the identity $\log_a\left(\frac{1}{x}\right) = -\log_a x$ for any base a and any positive value of x.

Strand: Patterns and Relations (Variables and Equations)
Students will:

- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes | **Illustrative Examples [Discretionary]**

P8–1. Distinguish between degree and radian measure, and solve problems, using both. [CN, E]

1.1 Draw an angle of one radian, and show how its radius and arc length are related.

1.2 Convert the following angles to degrees: $\frac{2\pi}{3}$, 1.6 rad.

1.3 Convert the following angles to radians expressed in terms of π: 180°, 55°.

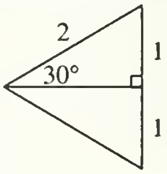
1.4 In an experiment to verify the law of refraction, measurements were made of the angles of incidence and refraction. A spreadsheet was used to calculate the sines of both angles and the ratio of the two sines. The results of the spreadsheet calculation are shown in the following table.

Angle of incidence i (degrees)	Angle of refraction r (degrees)	sin i	sin r	(sin i)/(sin r)
10	7	–0.544	0.657	–0.83
20	13	0.913	0.420	2.17
30	19	–0.988	0.150	–6.59
40	25	0.745	–0.132	–5.63
50	30	–0.262	–0.988	0.27
60	35	–0.305	–0.428	0.71
70	38	0.774	0.296	2.61
80	40	–0.994	0.745	–1.33

a) In the calculations of sin i and sin r, are the angles taken as being in degrees or in radians?
b) Modify the spreadsheet so that all entries reflect radian measure.
c) Modify the spreadsheet so that all entries reflect degree measure.
d) What conclusion can be drawn from either b) or c)?

P8–2. Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$. [CN, E]

2.1 Given an equilateral triangle with a side of 2 units, determine the exact trigonometric ratios of 30°.



2.2 Find the exact values for $\sin\frac{7\pi}{6}$, $\tan\frac{2\pi}{3}$, $\cos\frac{7\pi}{4}$.

Strand: Patterns and Relations (Variables and Equations)
Students will:
- represent algebraic expressions in multiple ways.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P8–3. Solve first and second degree trigonometric equations over a domain of length 2π: • algebraically • graphically. [PS, T]	3.1 Find, algebraically and graphically, the solution to the following trigonometric equations: a) $1 + 2\cos x = 5\cos x$; $0 \leq x < 2\pi$. Give solutions in decimal form. b) $\sin^2 x - \sin x = 0$; $0 \leq x < 2\pi$. Give solutions as exact values. c) $\cos 4x = 0.5$; $0 \leq x < 2\pi$. Give solutions as exact values.
P8–4. Determine the general solutions to trigonometric equations where the domain is the set of real numbers. [PS, T]	4.1 Sketch the graph of $y = \sin 3x$. Use the graph to find all solutions of $\sin 3x = 0$ in the interval $0 \leq x < 2\pi$. 4.2 Use technology to graph $y = x - 2\sin x$, and use the graph to find all solutions to the equation $2\sin x = x$. Express answers to a three-decimal place accuracy. 4.3 What is the relation between the graphs of $y = \sin x$ and $y = \frac{1}{2}$ and the roots of the equation $0 = 2\sin x - 1$? 4.4 Use technology to solve $\sin 3x = \frac{1}{2}$, and then write the general solution.
P8–5. Verify trigonometric identities: • numerically for any particular case • algebraically for general cases • graphically. [PS, R, T, V]	5.1 a) Verify that $\sin^2 x + \cos^2 x = 1$ for any real number x. b) Use this identity to show that $1 + \tan^2 x = \sec^2 x$ for any real number x, where $\cos x \neq 0$. 5.2 Given the identity $\frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$: a) verify the identity for the particular case when $x = \frac{\pi}{3}$ b) verify for a general angle, using an algebraic approach c) verify, by graphing the left-hand side and the right-hand side of the given identity.
P8–6. Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions. [R, T]	6.1 Write 2 (sin 5)(cos 5) in terms of a single trigonometric function. 6.2 Graph the function $f(x) = \frac{2\tan x}{1+\tan^2 x}$. a) Make a conjecture for the period of the above graph. b) Simplify the expression for $f(x)$ to a single trigonometric function, and then find the period of $f(x)$. c) Compare the solutions to a) and b).

Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Represent and analyze exponential and logarithmic functions, using technology as appropriate.	
Specific Outcomes	
P6–6. Graph and analyze an exponential function, using technology. [R, T, V]	6.1 Graph $y = 2^x$ with/without technology. 6.2 Graph $y = 4(2^x)$ and $y = 2^x$ on the same set of coordinate axes. Identify the domain, range, asymptotes and intercepts of each graph. What is the relationship between the two graphs?
P6–7. Model, graph and apply exponential functions to solve problems. [PS, T, V]	7.1 The summertime population of gophers in a field can be modelled by the equation $P = 100(1.1)^n$, where n is measured in years. Plot the graph for a 10-summer period, and use the graph to find out how long it takes for the gopher population to double. 7.2 The half-life of sodium-24 is 14.9 hours. Suppose that a hospital buys a 40 mg sample of sodium-24. a) How much of the sample will remain after 48 hours? b) How long will it be until only 1 mg remains? 7.3 The population of a certain country is 28 million and grows at a rate of 3% annually. Assuming the population is continuously growing, the population P, in millions, t years from now can be determined by the formula $P = 28e^{0.03t}$. a) In how many years will the population be 40 million? b) What factors could contribute to the breakdown of this model?
P6–8. Change functions from exponential form to logarithmic form and vice versa. [CN]	8.1 Rewrite $y = 2^x$ as a logarithmic function. 8.2 The ionization of pure water is shown in the equations: $[H^+]\,[OH^-] = 1.0 \times 10^{-14}$ and $[H^+] = [OH^-]$. If the pH of any solution is defined as $\text{pH} = -\log_{10}[H^+]$, what is the pH of pure water?
P6–9. Use logarithms to model practical problems. [CN, PS, V]	9.1 Research the strength of earthquakes, and compare them, using the Richter scale. 9.2 The Armenian earthquake, Richter scale 6.9, produced 3.5×10^{13} J of energy. How much energy did the Alaska earthquake, Richter scale 8.2, produce?

Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	**Illustrative Examples [Discretionary]**
P6–10. Explain the relationship between the laws of logarithms and the laws of exponents. [C, T]	10.1 Explain how the exponent law $a^x \times a^y = a^{(x+y)}$ is related to the logarithmic law $\log_a(MN) = \log_a M + \log_a N$. 10.2 Use a calculator to find $\log_5 8$, and justify your procedure.
P6–11. Graph and analyze logarithmic functions with and without technology. [R, T, V]	11.1 Graph $y = \log_{10} x$ and $y = \log_2 x$ on the same set of coordinate axes. What is the likely position of the graph of $y = \log_5 x$? 11.2 Analyze the graph of $y = \log_{10}(2x + 3)$. Identify the domain, range, asymptotes and intercepts.
General Outcome Represent and analyze trigonometric functions, using technology as appropriate. **Specific Outcomes**	
P8–7. Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position. [PS, R, V]	7.1 Triangle OBA has vertices $O(0, 0)$, $B(4, 0)$ and $A(4, 3)$. The unit circle, centred at (0, 0), intersects OA at point P. a) Use similar triangles to find the coordinates of point P. b) Use trigonometric ratios to find the sine and cosine of angle AOB. c) Compare your results in b) to the coordinates of point P.
P8–8. Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for: • amplitude, if defined • period • domain and range • asymptotes, if any • behaviour under transformations. [CN, T, V]	8.1 Using a graphing utility, graph $y = \sin x$ and $y = \cos x$ on the same set of axes. a) What relationship seems to exist between the two? b) What is the amplitude and period of each graph? 8.2 Graph $y = \tan x$ and $y = \tan 2x$. Compare the period, the domain and the range of $y = \tan x$ to those of $y = \tan 2x$. 8.3 In the equation $y = A \sin[B(x + C)] + D$; $A = 4$, $B = 3$, $C = \frac{-3\pi}{4}$ and $D = -3$. Compare the graph of this function to the graph of $y = \sin x$ with respect to domain, range, amplitude, period, x and y intercepts, horizontal phase shift and vertical displacement.

Strand: Patterns and Relations (Relations and Functions)
Students will:
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P8–9. Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for: • period • domain and range • asymptotes • behaviour under transformations. [CN, T, V]	9.1 Graph and analyze: a) $y = \sec x$ b) $y = \csc x$ c) $y = \cot x$. 9.2 Compare the domain, range and period of: a) $f(x) = \csc x$ and $g(x) = 5 \csc x$ b) $f(x) = \cot x$ and $g(x) = \cot 2x$.
P8–10. Use trigonometric functions to model and solve problems. [PS, R, V]	10.1 For a Saskatchewan town, the latest sunrise time is on December 21, at 09:15. The earliest sunrise time is on June 21, at 03:15. Sunrise times on other dates can be predicted from a sinusoidal equation. **Note**: There is no Daylight Saving Time in Saskatchewan. a) What is the equation that describes sunrise times? b) What is the amplitude and period of the equation describing sunrise times? c) Use the equation to predict the time of sunrise on April 9. d) What is the average time of sunrise throughout the year? 10.2 The depth of water in a harbour is given by the equation $d(t) = -4.5 \cos (0.16\pi t) + 13.7$, where $d(t)$ is the depth, in metres, and t is the time, in hours, after low tide. a) Sketch the graph of $d(t)$. b) What is the period of the tide, from one high tide to the next? c) A bulk carrier needs at least 14.5 m of water to dock safely. For how many hours per cycle can the bulk carrier dock safely? 10.3 The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 23.6°C on July 26, and a lowest value of 4.2°C on January 26. a) Describe this variation with a sine or cosine equation. b) What is the expected maximum temperature for May 26? c) How many days will have an expected maximum of 21.0°C or higher? d) Explain why different equations give the same answers for b) and c).

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Classify conic sections, using their shapes and equations.	
Specific Outcomes	
P9–1. Classify conic sections according to shape. [C, R, V]	1.1 Visualize the shapes generated from the intersection of a double-napped cone and a plane. For each conic section, describe the relationship between the plane, the central axis of the cone and the cone's generator.
P9–2. Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only). [CN, T, V]	2.1 A circle with a radius of 4 units has the equation $x^2 + y^2 - 16 = 0$. What are the values of A, C and F in the general form? What is the radius of the circle $25x^2 + 25y^2 - 100 = 0$? 2.2 a) Graph the circle $x^2 + y^2 = 25$. b) Graph $Ax^2 + y^2 = 25$ where $A > 1$. c) Graph $Ax^2 + y^2 = 25$ where $0 < A < 1$. d) Graph $Ax^2 + y^2 = 25$ where $A = 0$. e) Graph $x^2 + Cy^2 = 25$ where $C > 1$. f) Graph $x^2 + Cy^2 = 25$ where $0 < C < 1$. g) Graph $x^2 + Cy^2 = 25$ where $C = 0$. h) Draw a conclusion based on the results found in b) through g). 2.3 Graph $2x^2 + y^2 - 12 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph? 2.4 Graph $4x^2 - 25y^2 - 100 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?
P9–3. Convert a given equation of a conic section from general to standard form and vice versa. [R, T]	3.1 Convert to standard form: a) $x^2 + y^2 + 6x - 8y = 11$ b) $3x^2 + y^2 + 6x + 4y = 9$. 3.2 Convert to general form: a) $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{16} = 1$ b) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{16} = 1$.

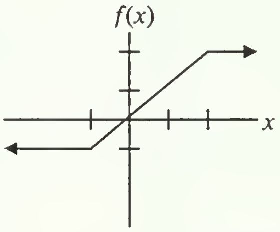
Strand: Shape and Space (Transformations)
Students will:
- perform, analyze and create transformations.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Perform, analyze and create transformations of functions and relations that are described by equations or graphs.	
Specific Outcomes	
P9–4. Describe how various translations of functions affect graphs and their related equations: • $y = f(x - h)$ • $y - k = f(x)$. [C, T, V]	4.1 Describe how the graph of $y = x^2$ compares to the graph of $y = x^2 - 2$. 4.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $f(x) - 2$ b) $f(x - 2)$ c) $f(x - 2) + 1$.
P9–5. Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: • $y = af(x)$ • $y = f(kx)$. [C, T, V]	5.1 Describe how the graph of $y = x^2$ compares to the graph of: a) $y = 2x^2$ b) $y = \frac{2}{3}x^2$. 5.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of: a) $2f(x)$ b) $-2f(x)$ c) $\frac{2}{3}f(x)$. Discuss the changes. 5.3 Given the graph of $f(x) = \sin x$, sketch the graph of: a) $f(2x)$ b) $\frac{2}{3}f(x)$. 5.4 Given the graph of $f(x) = x^3$ and its image under the transformation $g(x) = 3f(x)$, find the equation describing $g(x)$.

Strand: Shape and Space (Transformations)
Students will:
- perform, analyze and create transformations.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P9–6. Describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$. [C, T, V]	6.1 Graph any function $f(x)$. Sketch the graph of: a) $-f(x)$ b) $f(-x)$ c) $f^{-1}(x)$ d) $f^{-1}[f(x)]$. 6.2 If $g(x)$ is the reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.
P9–7. Using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$. [C, T, V]	7.1 Given $f(x) = 2x + 1$, sketch the graph of $f(x)$ and of $\frac{1}{f(x)}$. What happens to the x-intercepts of $f(x)$? 7.2 Sketch the graph of $f(x) = \sin x$, and sketch $\frac{1}{\sin x}$. 7.3 Sketch $\frac{1}{f(x)}$, if $f(x)$ is shown by the accompanying sketch. 

Strand: Shape and Space (Transformations)
Students will:
- perform, analyze and create transformations.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P9–8. Using the graph and/or the equation of $f(x)$, describe and sketch $\lvert f(x)\rvert$. [C, T, V]	8.1 Given the graph of $f(x) = 2x + 1$, sketch $\lvert f(x)\rvert$. 8.2 Sketch $y = 3\lvert\sin x\rvert$. What is the period of this function? 8.3 Sketch $f(x) = \dfrac{1}{\lvert x^2 - 1\rvert}$. 8.4 An AC generator has a voltage given by $V = 170\cos(120\pi t)$, where V is the voltage and t the time in seconds. A simple DC rectifier has voltage output given by $V = 170\lvert\cos(120\pi t)\rvert$. Sketch the output graphs for both devices, and describe the similarities and differences.
P9–9. Describe and perform single transformations and combinations of transformations on functions and relations. [C, T, V]	9.1 Given $f(x) = x^2$, sketch the graph of $f(x)$, and sketch the graph of $-2f(x - 1) + 3$. 9.2 Determine the equation of the ellipse $x^2 + 4y^2 - 25 = 0$, after each of the following transformations: a) translated two units to the right b) translated three units down c) expanded by a factor of two along the horizontal axis d) expanded by a factor of one quarter along the vertical axis. 9.3 Given the circle $x^2 + y^2 = 1$ and its image under a translation described by the ordered pair $(2, -3)$: a) write the equation of the image b) if a point $P(a, b)$ is on the graph of the circle $x^2 + y^2 = 1$ and $P'(a', b')$ is the transformed image of P, what are the coordinates of P' in terms of a and b?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes | **Illustrative Examples [Discretionary]**

General Outcome

Use normal and binomial probability distributions to solve problems involving uncertainty.

Specific Outcomes

C6–1. Find the population standard deviation of a data set or a probability distribution, using technology. [CN, E, T, V]

1.1 Measure the height of each student in a class, and calculate the mean and standard deviation.

1.2 A company uses an automated packaging device to produce 50-g bags of Karmel Korn. The machine needs frequent checking to see if it is actually putting 50 g in each bag. The following are the masses, in grams, of thirty bags of Karmel Korn.

54	50	47	50	51	50
53	50	47	51	50	51
52	49	46	52	50	49
52	48	48	53	49	49
51	48	49	52	49	50

a) Calculate the mean and standard deviation of this data.
b) What problems will be encountered, if the standard deviation gets too high?

Dottori et al., *Foundations of Mathematics 11*, p. 391. Adapted with permission.

C6–2. Use z-scores and z-score tables to solve problems. [PS, R, T, V]

2.1 The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL.

a) Calculate the z-score for a can with a volume of 355 mL.
b) What percentage of production will consist of cans having content volumes between 350 mL and 355 mL?
c) What percentage of production will consist of cans having content volumes less than 355 mL?
d) If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected in a run of 50 000?

Strand: Statistics and Probability (Chance and Uncertainty)

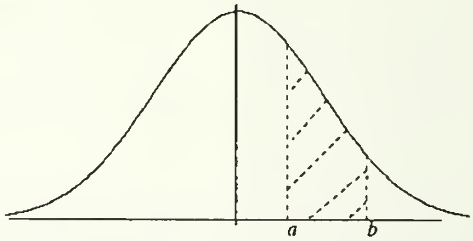
Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]

2.2



a) What is the area under this curve?
b) If $P(a < z < b) = 0.4$, what is the area under the curve for the interval $a < z < b$?
c) If $P(z < b) = 0.9$, calculate $P(z > b)$, and calculate the value of b.

2.3 For entry into the Canadian Armed Forces, the standards for height used to be set at 158 cm to 194 cm for males, and 152 cm to 184 cm for females. Use the concept of z-score to test if these two height standards are equivalent. Assume means of 176 cm and 163 cm and standard deviations of 8 cm and 7 cm respectively.

2.4 A sample of 122 people gives a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find:
a) the expected number of people with temperatures above 37.0°C
b) the expected number of people with temperatures below 36.0°C.

Also, estimate the range of temperatures contained within the sample.

2.5 In the general population, the IQ scores of individuals is normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested:
a) What proportion of this group is expected to have IQs between 100 and 120?
b) What is the probability that an individual in the group has an IQ greater than 120?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:
- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

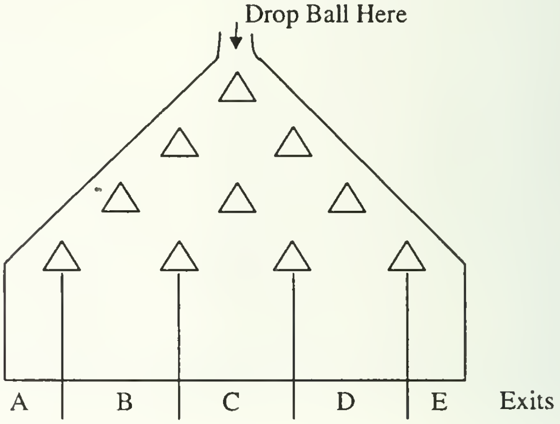
General and Specific Outcomes	Illustrative Examples [Discretionary]
C6–3. Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS]	3.1 The heights of males employed by a manufacturer follow a normal distribution with a mean of 169 cm and a standard deviation of 8 cm. a) Establish a symmetric 95% confidence interval for the average height in a random sample of 36 male employees. b) What happens to the width of the symmetric 95% confidence interval, if the sample size is increased from 36 to 225? 3.2 Pollsters estimate that the number of decided voters in favour of a particular bylaw is 64%, and the number opposed is 36%. a) If the sample size is 250, find the expected mean and standard deviation of *yes* voters. b) Estimate, for this sample, the expected percentage of *yes* voters, with a symmetric 95% confidence interval used to establish the margin of error. c) If the margin of error for the percentage of *yes* voters must be less than ±1.0%, what would be the minimum sample size required? 3.3 The probability that a car salesperson will complete a sale is 0.10. If the salesperson has 200 customers in the next month, establish a symmetric 95% confidence interval for the number of completed sales for the month.

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. **Specific Outcomes**	
C6–4. Solve pathway problems, interpreting and applying any constraints. [PS, R]	4.1 Given the following "pinball" situation, what is the probability of the ball reaching each of the exits?  What assumptions are made in the solution?
C6–5. Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]	5.1 Joe has three different shirts, two different pairs of pants and five different pairs of shoes. List all possible outfits in such a way as to ensure that all have been counted and none have been counted twice. How many possible outfits are there? Use the fundamental counting principle to determine the number of outfits there should be. Do your answers match? 5.2 An airline pilot reported that in seven days she spent one day in Winnipeg, one day in Regina, two days in Edmonton and three days in Yellowknife. How many different itineraries are possible? What difference would it make if the first day and the last day had to be spent in Yellowknife?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

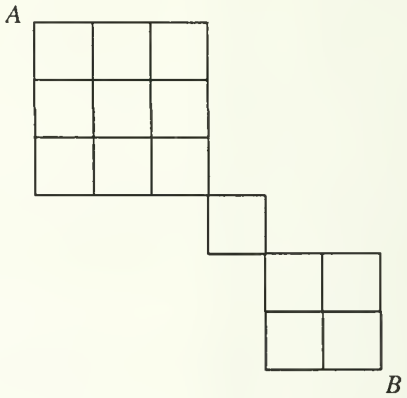
General and Specific Outcomes	Illustrative Examples [Discretionary]
P7–12. Determine the number of permutations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	12.1 List all possible permutations of the letters in the word **bold**. 12.2 Calculate the number of ways that an executive consisting of four people (president, vice-president, treasurer and secretary) can be selected from a group of 20 people. 12.3 Explain the meaning of ${}_8P_3$. Why does ${}_3P_8$ not make sense? 12.4 Develop and solve a problem where ${}_8P_3$ would be applicable. 12.5 Solve ${}_nP_2 = 30$. 12.6 On a 12-question multiple-choice test, three answers are A, three are B, three are C and three are D. How many different answer keys are possible?
P7–13. Determine the number of combinations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]	13.1 From a group of five student representatives, three will be chosen to work on the dance committee. a) List all possible committees. b) Calculate ${}_5C_3$, and compare to the answer in part a). c) If the committee had to have a chairperson, would it still be a combination? Why or why not? d) How many committees of three, with a chairperson, can be chosen from a group of 10 student representatives? 13.2 Show that ${}_nC_k = {}_nC_{(n-k)}$, using two different methods. Verify the truth of this assertion for the special case with $n = 10$ and $k = 3$. 13.3 How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an n-sided polygon?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P7–14. Determine the number of pathways in a given compound pathway problem. [CN, PS, V]	14.1 Student *A* wants to visit Student *B*. Roads are shown as lines on the grid. Only south and east travel directions can be used.  a) How many different paths can *A* take to get to *B*, if *A* has to travel along the lines that represent the roads? b) Change the middle square to a 2 × 2 grid, and repeat the question.
P7–15. Solve problems, using the binomial theorem where *N* belongs to the set of natural numbers. [CN, E, V]	15.1 Expand $(x + y)^7$, using the binomial theorem. 15.2 Find the 11th term of the expansion of $(x - 2)^{13}$. 15.3 Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins . . . , and make an organized list. Relate this organized list to Pascal's triangle and the binomial theorem. 15.4 Given a set of four elements, list the different proper and improper subsets, and organize them. How is this related to Pascal's triangle? How many subsets are there in total?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
General Outcome Model the probability of a compound event, and solve problems based on the combining of simpler probabilities. **Specific Outcomes**	
C6–6. Construct a sample space for two or three events. [PS, R, V]	6.1 List the sample space for rolling a 6-sided die and flipping a coin. 6.2 Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15 inclusive. A train is scheduled to arrive between 07:11 and 07:17 inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the point (6, 14). Times are expressed in whole minutes. a) How many points are there in this sample space? b) How many points have the bus and the train arriving at the same time? c) How many points have the bus arriving after the train? d) What is the probability of the bus arriving after the train?
C6–7. Classify events as independent or dependent. [C]	7.1 Classify the following events as independent or dependent: a) tossing a head in a coin toss and rolling a 6 on a die b) drawing an ace for the first card and another ace for the second, if the experiment is carried out without replacement c) drawing a king for the first card and a queen for the second, if the experiment is carried out with replacement. 7.2 Sixty per cent of young drivers take driver training, and 25% of young drivers have an accident in their first year of driving. Statistics show that 10% of those who do take driver training have an accident in their first year. Are taking driver training and having an accident in the first year independent events?

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
C6–8. Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]	8.1 If the probability of winning a game is $\frac{1}{31}$, what is the probability of losing the game? 8.2 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot?
P7–16. Determine the conditional probability of two events (Bayes' law). [E, PS, R]	16.1 In a particular country, the probability of a child being a girl is 0.510. A family of five children is known to have at least two girls. What is the probability of this family having exactly four girls? 16.2 It is known that 10% of a population has a certain disease. For a patient without the disease, a blood test for the disease gives a correct diagnosis 95% of the time. For a patient with the disease, the test gives a correct diagnosis 99% of the time. What is the probability that a person whose blood test shows the disease actually has the disease?
P7–17. Solve probability problems involving permutations, combinations and conditional probability. [E, PS, R]	17.1 Five books, each of a different colour, and including one red and one green book, are placed on a shelf. What is the probability of the red book being at one end and the green book at the other? 17.2 What is the probability of holding all four aces in a five-card hand dealt from a standard 52-card deck? 17.3 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot. a) If Team A shoots first, what is the probability of Team B winning on its first shot? b) If Team A shoots first, what is the probability of Team A winning on its third shot? c) What is the probability of Team A eventually winning? d) If Team B shot first, what is the probability of Team B eventually winning?

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[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	Illustrative Examples [Discretionary]
P7–18. Solve probability problems, using the binomial distribution as applied to small samples. [PS, R, T]	18.1 A written test for a driver's licence consists of 10 multiple-choice questions. To pass the test, a driver must answer 9 or 10 questions correctly. What is the probability of passing by guessing, if there are four possible answers to each question? 18.2 A family has nine children. Assuming that there is an equal likelihood for male and female births, what is the probability that there are seven boys and two girls? 18.3 An 8 km/h crash test was given to a sample of 20 cars. Four cars failed the test because of damaged bumpers. Find a 95% confidence interval for the proportion of cars that would fail this crash test.

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